On flow-aware CSMA

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Performance 2010

Outline

Model

Background

Standard CSMA

Flow-aware CSMA

Conclusion

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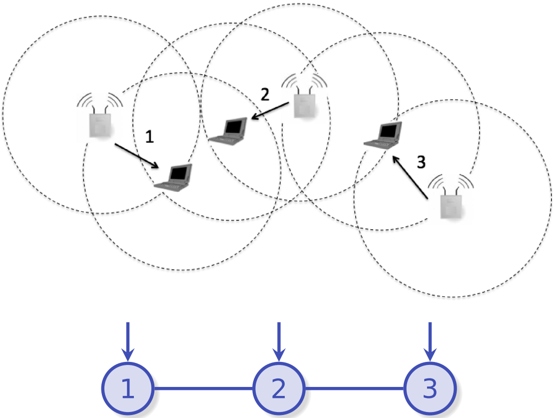
Conflict graph

The network is represented by a conflict graph



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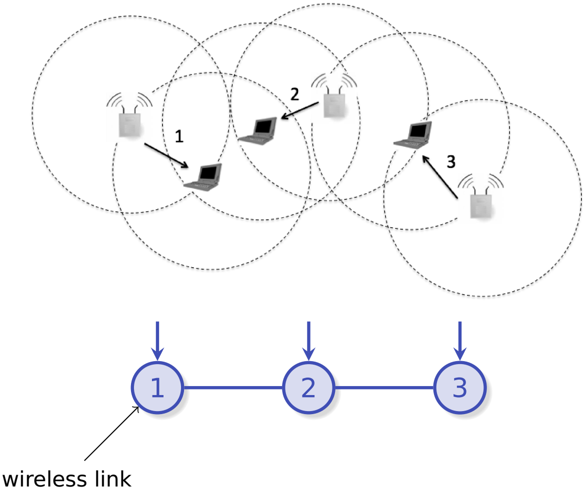

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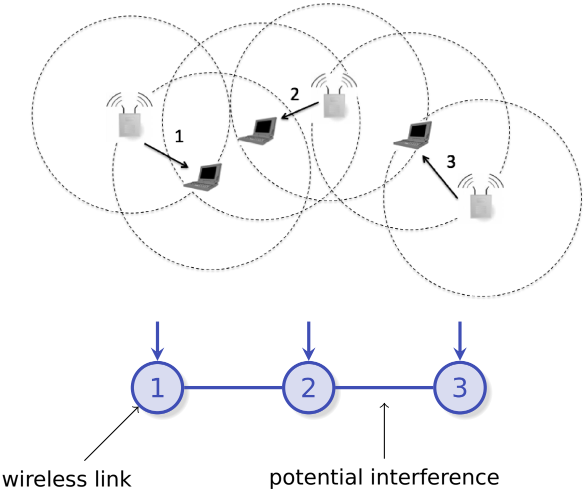

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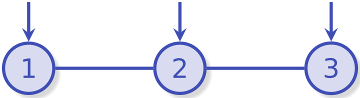
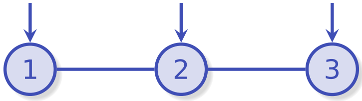


Schedules:

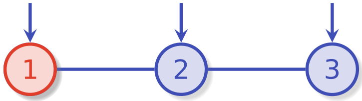
Conflict graph



Schedules:

- $\varnothing$,

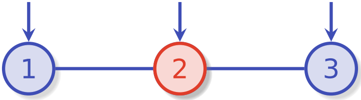
Conflict graph



Schedules:

- $\emptyset$,
- $\{1\}$,

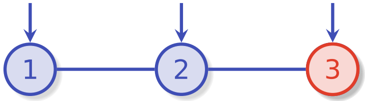
Conflict graph



Schedules:

- $\emptyset$,
- $\{1\}$,
- $\{2\}$,

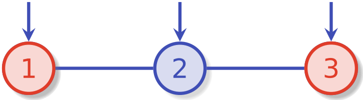
Conflict graph



Schedules:

- ∅,
- {1},
- {2},
- {3},

Conflict graph



Schedules:

- $\emptyset$,
- $\{1\}$,
- $\{2\}$,
- $\{3\}$,
- $\{1, 3\}$.

Time-scale separation

We suppose time-scale separation between flows and network access.

- For the network access algorithm, the flows are fixed.
- For the flows, network is at average : the throughput of a node is the fraction of time it is active.

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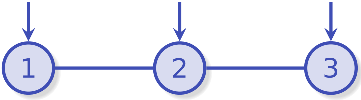
Example:



$$\phi_1 = p_{\{1\}} + p_{\{1,3\}}, \ \phi_2 = p_{\{2\}}, \ \phi_3 = p_{\{3\}} + p_{\{1,3\}}.$$

Capacity Region

Defined as the set of all feasible link throughputs

$$\phi_k = \sum_{i: k \in S_i} p_i$$

Throughput of node *k*

Schedule *i*

Probability of schedule *i*

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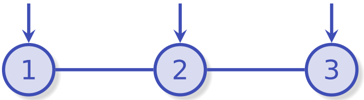


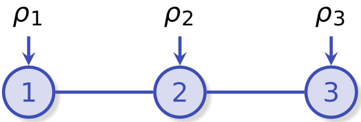
Schedules: $\varnothing$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 3\}$.
Capacity region: $\{\phi_1 + \phi_2 \leq 1,\ \phi_2 + \phi_3 \leq 1\}$.

Stability region

Defined as the set of traffic intensities such that the network is stable.

$$\rho_k = \lambda_k \times \sigma_k$$

Example:



The stability region depends on the algorithm.

Stability region

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↑

Flows arrival rate

Stability region

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Mean flow size

Flows arrival rate

Stability region

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$$\rho_k = \lambda_k \times \sigma_k$$

Traffic intensity (ρ_k)

Flows arrival rate (λ_k)

Mean flow size (σ_k)

Optimal stability region

The stability region of any algorithm is included in the capacity region. The interior of the capacity region is called the optimal stability region.

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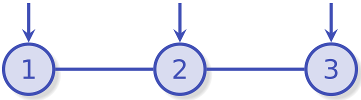
Example:



Optimal stability region:

$$\{\rho_1 + \rho_2 < 1,\ \rho_2 + \rho_3 < 1\}.$$

Outline

Maximal Weight scheduling

Tassiulas & Ephremides 92

$$\max w_i(x) = \sum_{k \in S_i} x_k$$

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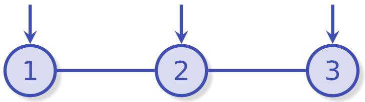
Example:



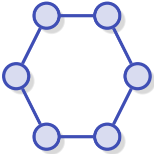
$$x_1 + x_3 > x_2 \Rightarrow \text{schedule } \{1, 3\}$$
$$x_1 + x_3 < x_2 \Rightarrow \text{schedule } \{2\}$$

Suboptimal algorithms

Maximal queue scheduling

Mc Keown 95

$\max x_k$



Dimakis & Walrand 06
Efficiency $\frac{8}{9} \approx 0.89$

Suboptimal algorithms

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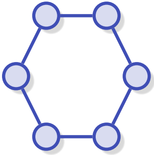

Dimakis & Walrand 06
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Maximal size scheduling

Charporkar, Kar & Sarkar 95

$\max |S_i|$

Bonald & Massoulié 01
Efficiency ≈ 0.76

Optimal Algorithms

Adaptative rate-based CSMA

Jiang & Walrand 08

- Measure the packets input rate and the output rate and adapt the back-off accordingly.
- Learning algorithm.

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Adaptative rate-based CSMA

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- Learning algorithm.

Adaptative queue based CSMA

Rajagopolan, Shah & Shin 09

- Adapt the back-off according to the $\log\log$ of the queue length.
- Some technical issues

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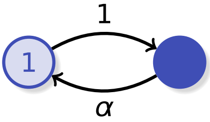
Standard CSMA

- Exponential backoff times
- Corresponds to a Loss network



Back-off	Transmission	Back-off
$\sim \exp(\alpha)$	$\sim \exp(1)$	$\sim \exp(\alpha)$

$$\phi_1(x) = \frac{\alpha}{1+\alpha}$$

Standard CSMA

- Exponential backoff times
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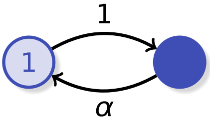
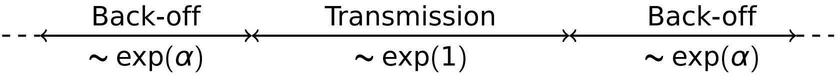




$$\phi_1(x) = \frac{\alpha}{1+\alpha}$$

Stability region: $\left\{\rho_1 < \frac{\alpha}{1+\alpha}\right\} \xrightarrow{\alpha\to\infty} \{\rho_1 < 1\}$

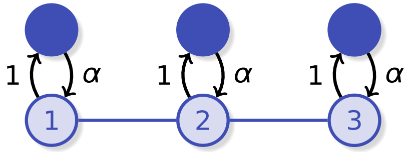
Standard CSMA

$$w_i(x) = \prod_{k \in S_i} \alpha_k \mathbb{1}_{\{x_k > 0\}}$$

Weight of schedule i

Ratio of transmission time to backoff time at link k

Example: Assume $\alpha_k = \alpha$ and $x_k > 0$ for $k = 1, 2, 3$.



$$\emptyset : \; w_i(x) = 1, \; \{1\}, \{2\}, \{3\} : \; w_i(x) = \alpha,$$

$$\{1, 3\} : \; w_i(x) = \alpha^2.$$

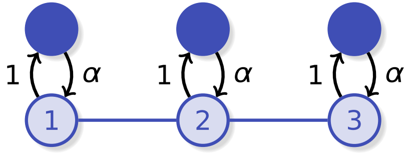
Standard CSMA

$$\phi_k(x) = \sum_{i:k \in S_i} p_i(x)$$

Throughput of link k

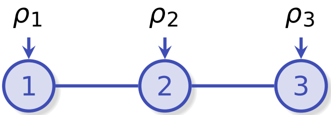
Probability of schedule i

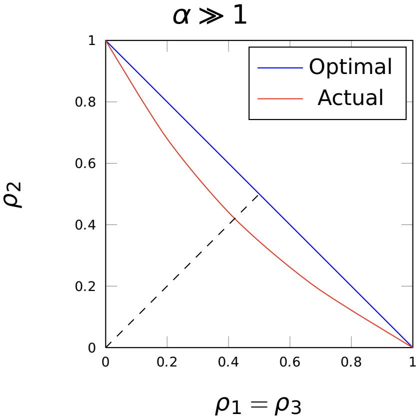
Example: Assume $\alpha_k = \alpha$ and $x_k > 0$ for $k = 1, 2, 3$.



$$\phi_1(x) = \phi_3(x) = \frac{\alpha^2 + \alpha}{1 + 3\alpha + \alpha^2}, \;\; \phi_2(x) = \frac{\alpha}{1 + 3\alpha + \alpha^2}.$$

Suboptimality of standard CSMA





Efficiency ≈ 0.84.

Idea of the proof:
When $\alpha \gg 1$, $x_1 > 0$ and $x_2 > 0$,

$$\phi_2(x) \approx 0.$$

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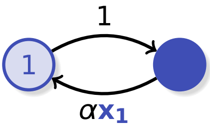
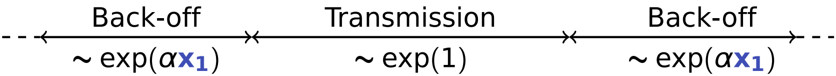
Flow-aware CSMA

Proposed modification of CSMA:

- Exponential backoff time for each flow
- Corresponds also to a Loss network





$$\phi_1(x) = \frac{\alpha \mathbf{x_1}}{1 + \alpha \mathbf{x_1}}$$

Flow-aware CSMA

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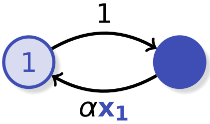
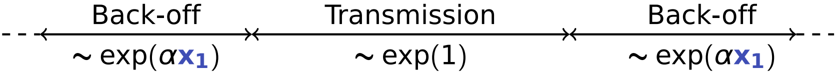




$$\phi_1(x) = \frac{\alpha \mathbf{x_1}}{1 + \alpha \mathbf{x_1}}$$

Stability region:

$$\{\rho_1 < 1\}$$

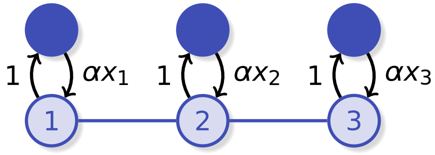
Flow-aware CSMA

$$w_i(x) = \prod_{k \in S_i} \alpha_k x_k$$

Weight of schedule i

Ratio of transmission time to backoff time at link k

Example: Assume $\alpha_k = \alpha$.



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Flow-aware CSMA

$$\phi_k(x) = \sum_{i:k \in S_i} p_i(x)$$

Throughput of link k

Probability of schedule i

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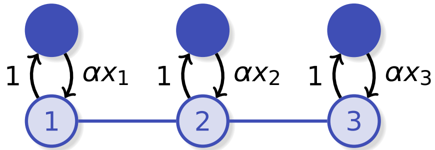


$$\phi_1(x) = \frac{\alpha^2 x_1 x_3 + \alpha x_1}{1 + \alpha(x_1 + x_2 + x3) + \alpha^2 x_1 x_3},$$

$$\phi_2(x) = \frac{\alpha x_2}{1 + \alpha(x_1 + x_2 + x_3) + \alpha^2 x_1 x_3}.$$

Optimality of flow-aware CSMA

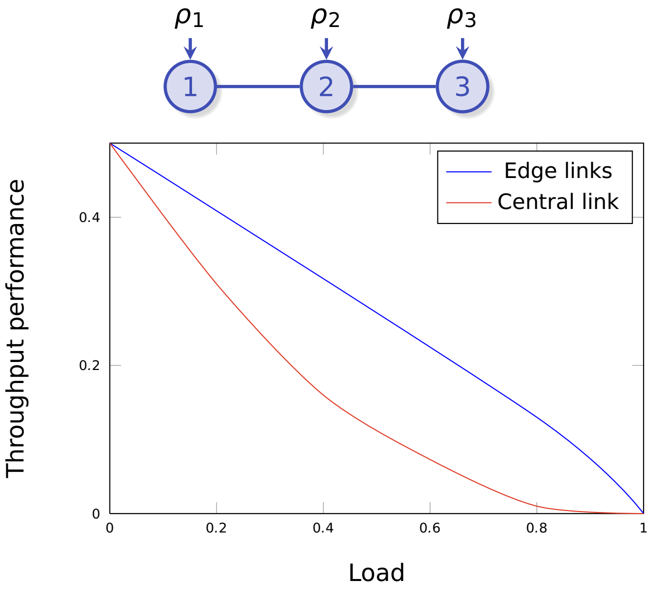
Theorem (**Main result**)

The flow-aware CSMA algorithm is optimal: it stabilizes the network for all traffic intensities inside the capacity region.

Sketch of proof:

- When the number of flows is high, the algorithm behaves as Max-weight: the algorithm selects schedules of high weight.
- Based on a Lyapunov function and Foster's criterion.

Throughput performance



Outline

Conclusion

A simple, distributed, asynchronous access scheme that is provably optimal.
This is still true in the multi-channel case.

A theoretical issue:

- Time-scale separation

Extensions of the model:

- Collisions
- Hidden nodes
- Throughput performance