On Flow-Aware CSMA

in Multi-Channel Wireless Networks

Mathieu Feuillet

Joint work with Thomas Bonald

CISS 2011

Outline

Model

Background

Standard CSMA

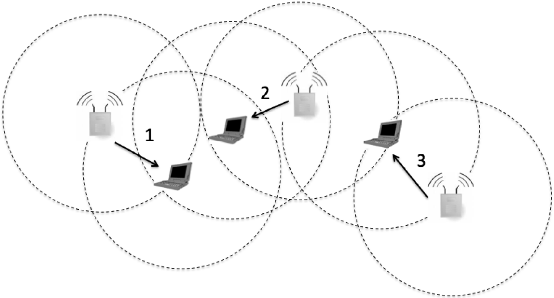
Flow-aware CSMA

Conclusion

Outline

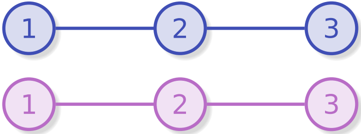
Conflict graphs

The network is represented by a set of conflict graphs, one per channel.



Channel 1

Channel 2



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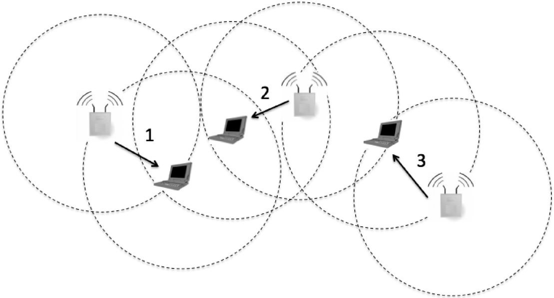

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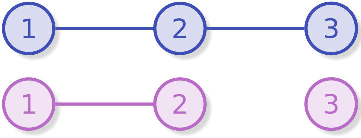
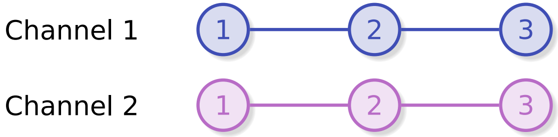


Conflict graphs on different channels can be different.

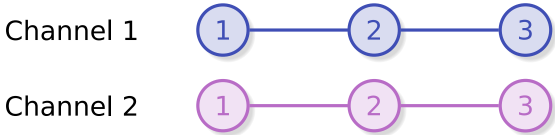
Conflict graphs



Schedules:

- $\emptyset$, $\{(1,1)\}$, $\{(2,1)\}$, $\{(3,1)\}$,$\{(1,1),(3,1)\}$.

Conflict graphs



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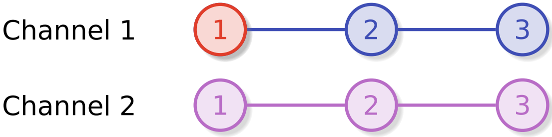

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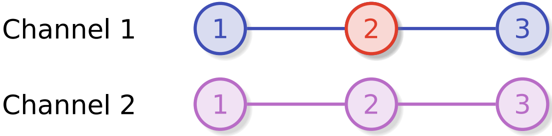

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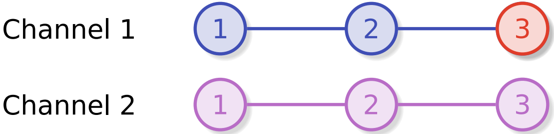

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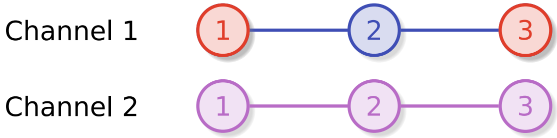

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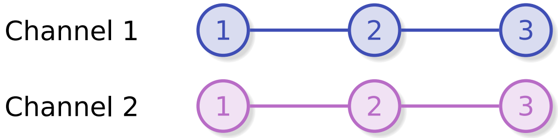

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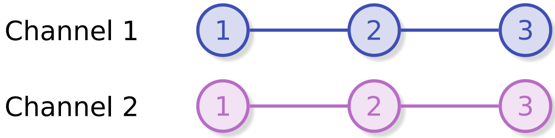

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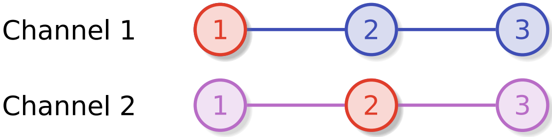

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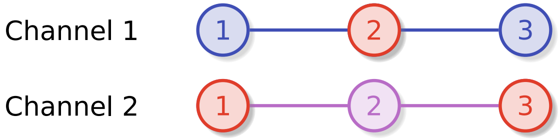

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Time-scale separation

We assume time-scale separation between flow-level and packet-level dynamics.

- At the packet level, the number of flows is fixed.
- At the flow level, the packet-level dynamics are at equilibrium: the throughput of each node is the fraction of time it is active.

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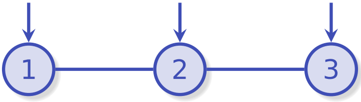
Example:



$$\phi_1 = p_{\{1\}} + p_{\{1,3\}}, \ \phi_2 = p_{\{2\}}, \ \phi_3 = p_{\{3\}} + p_{\{1,3\}}.$$

Capacity Region

Defined as the set of all feasible link throughputs

$$\phi_k = \sum_{i:k \in S_i} p_i$$

Throughput of node *k*

Schedule *i*

Probability of schedule *i*

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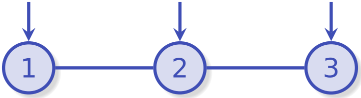


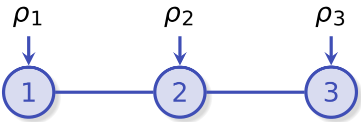
Schedules: $\emptyset$, $\{1\}$, $\{2\}$, $\{3\}$, $\{1,3\}$.
Capacity region: $\{\phi_1 + \phi_2 \leq 1,\ \phi_2 + \phi_3 \leq 1\}$.

Stability region

Defined as the set of traffic intensities such that the network is stable.

$$\rho_k = \lambda_k \times \sigma_k$$

Example:



The stability region depends on the algorithm.

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Flows arrival rate

Stability region

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Optimal stability region

The stability region of any algorithm is included in the capacity region. The interior of the capacity region is called the optimal stability region.

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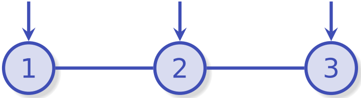

Optimal stability region:

$$\{\rho_1 + \rho_2 < 1,\ \rho_2 + \rho_3 < 1\}.$$

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Maximal Weight scheduling

Tassiulas & Ephremides 92

$$\max w_i(x) = \sum_{k \in S_i} x_k$$

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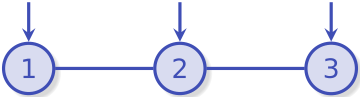


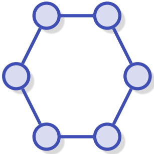
$$x_1 + x_3 > x_2 \Rightarrow \text{schedule } \{1, 3\}$$
$$x_1 + x_3 < x_2 \Rightarrow \text{schedule } \{2\}$$

Suboptimal algorithms

Maximal queue scheduling

Mc Keown 95

$\max x_k$



Dimakis & Walrand 06
Efficiency $\frac{8}{9} \approx 0.89$

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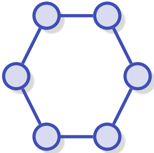

Dimakis & Walrand 06
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Maximal size scheduling

Charporkar, Kar & Sarkar 95

$\max |S_i|$

Bonald & Massoulié 01
Efficiency ≈ 0.76

Optimal Algorithms

Adaptative rate-based CSMA

Jiang & Walrand 08

- Measure the packet input and output rates and adapt the back-off accordingly.
- Learning algorithm.

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Adaptative queue based CSMA

Rajagopolan, Shah & Shin 09

- Adapt the back-off according to the $\log\log$ of the queue length.
- Some technical assumptions.

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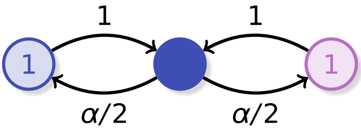
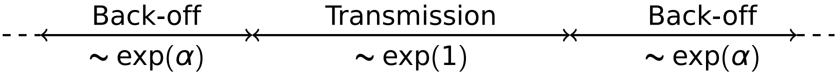
Standard CSMA

Probe a channel at random, after some random backoff time.





$$\phi_1(x) = \frac{\alpha}{1+\alpha}$$

Standard CSMA

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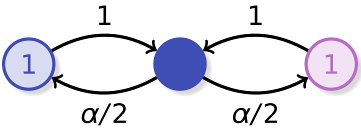
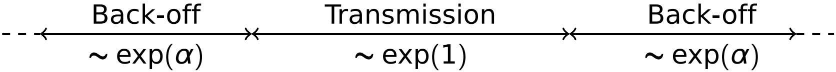




$$\phi_1(x) = \frac{\alpha}{1+\alpha}$$

Stability region: $\left\{\rho_1 < \frac{\alpha}{1+\alpha}\right\} \xrightarrow{\alpha\to\infty} \{\rho_1 < 1\}$

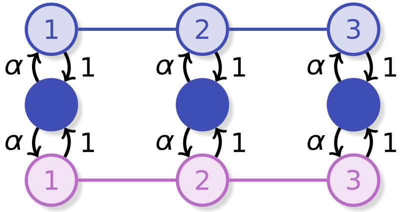
Standard CSMA

$$w_i(x) = \prod_{(k,j)\in S_i} \alpha_{k,j}{}^{\mathbb{1}\{x_k>0\}}$$

Weight of schedule i

Ratio of transmission time to virtual backoff time at link k on channel j

Example: Assume $\alpha_{k,j} = \alpha$ for $k = 1, 2, 3$ and $j = 1, 2$.



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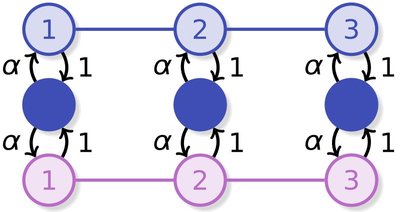

$$\varnothing : \quad w_i(x) = 1$$

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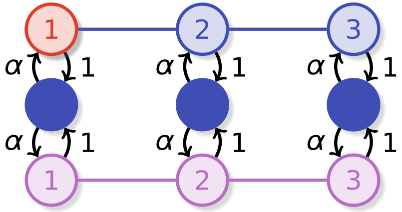

$$\{(1,1)\} : \quad w_i(x) = \alpha$$

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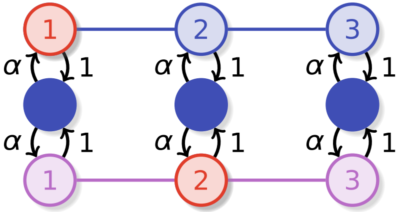


$$\{(1,1),(2,2)\}: \quad w_i(x) = \alpha^2$$

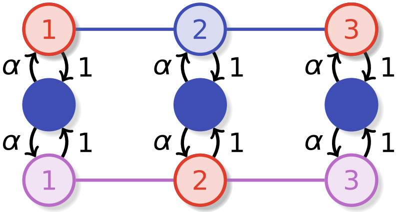
Standard CSMA

$$w_i(x) = \prod_{(k,j)\in S_i} \alpha_{k,j}{}^{\mathbb{1}_{\{x_k>0\}}}$$

Weight of schedule i

Ratio of transmission time to virtual backoff time at link k on channel j

Example: Assume $\alpha_{k,j} = \alpha$ for $k = 1, 2, 3$ and $j = 1, 2$.



$$\{(1,1),(2,2),(3,1)\}: \quad w_i(x) = \alpha^3$$

Standard CSMA

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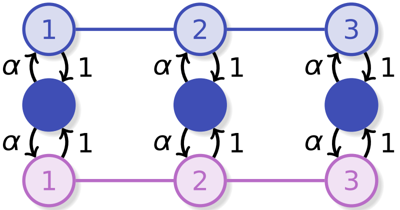


$w_i(x) = \alpha^n$ for n active links

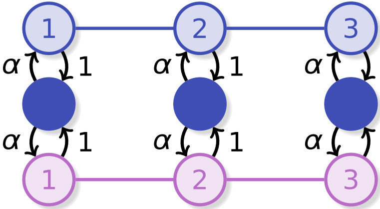
Standard CSMA

$$\phi_k(x) = \sum_{i: k \in S_i} p_i(x)$$

Throughput of link k

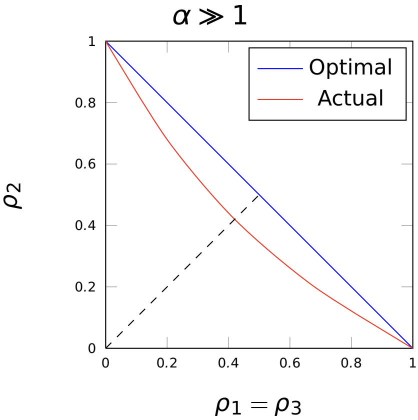
Probability of schedule i

Example: Assume $\alpha_{k,j} = \alpha$ for $k = 1, 2, 3$ and $j = 1, 2$.



$$\phi_1(x) = \phi_3(x) = \frac{2\alpha + 6\alpha^2 + 2\alpha^3}{1 + 6\alpha + 8\alpha^2 + 2\alpha^3}, \; \phi_2(x) = \frac{2\alpha + 4\alpha^2 + 2\alpha^3}{1 + 6\alpha + 8\alpha^2 + 2\alpha^3}$$

Suboptimality of standard CSMA
1 channel



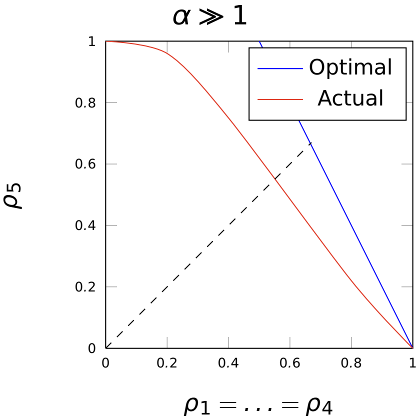
Homogeneous case

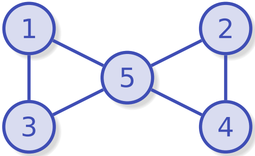
$$\rho_1 = \rho_2 = \rho_3$$

Efficiency ≈ 0.84.

Suboptimality of standard CSMA

2 channels





Homogeneous case

$\rho_1 = \ldots = \rho_5$

Efficiency ≈ 0.85

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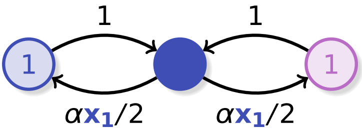
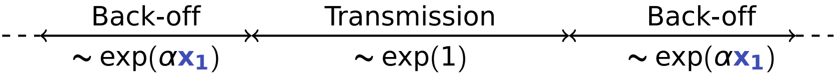
Flow-aware CSMA

Proposed modification of CSMA:
Exponential backoff time for each flow





$$\phi_1(x) = \frac{\alpha \mathbf{x_1}}{1 + \alpha \mathbf{x_1}}$$

Flow-aware CSMA

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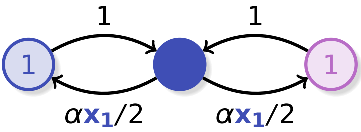

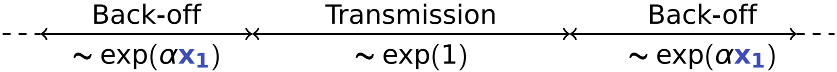

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Stability region: $\{\rho_1 < 1\}$

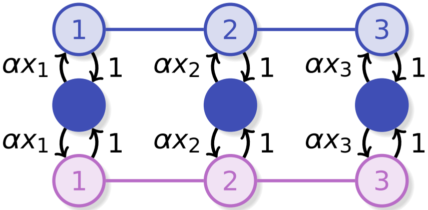
Flow-aware CSMA

$$w_i(x) = \prod_{k \in S_i} \alpha_{k,j} x_k$$

Weight of schedule i

Ratio of transmission time to virtual back-off time at link k on channel j

Example: Assume $\alpha_{k,j} = \alpha$ for $k = 1, 2, 3$ and $j = 1, 2$.



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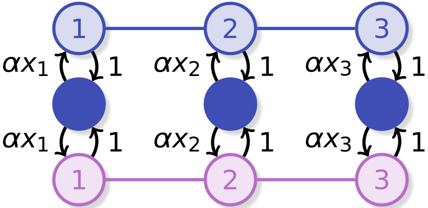

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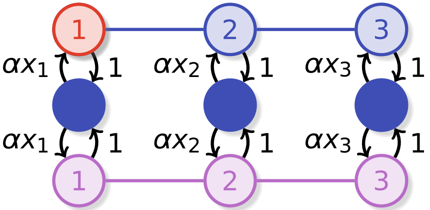

$$\{(1,1)\} : \quad w_i(x) = \alpha x_1$$

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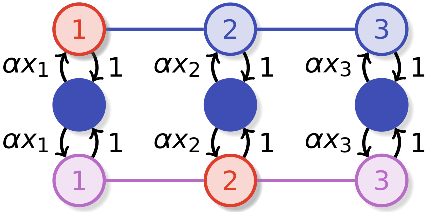
Example: Assume $\alpha_{k,j} = \alpha$ for $k = 1, 2, 3$ and $j = 1, 2$.



$$\{(1,1),(2,2)\}: \quad w_i(x) = \alpha^2 x_1 x_2$$

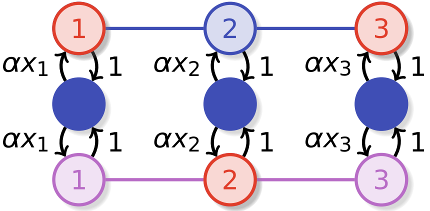
Flow-aware CSMA

$$w_i(x) = \prod_{k \in S_i} \alpha_{k,j} x_k$$

Weight of schedule i

Ratio of transmission time to virtual back-off time at link k on channel j

Example: Assume $\alpha_{k,j} = \alpha$ for $k = 1, 2, 3$ and $j = 1, 2$.



$$\{(1,1),(2,2),(3,1)\}: \quad w_i(x) = \alpha^3 x_1 x_2 x_3$$

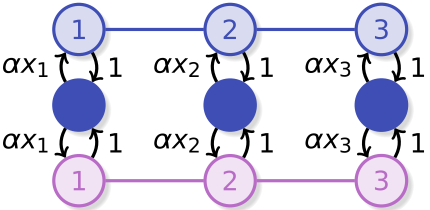
Flow-aware CSMA

$$\phi_k(x) = \sum_{i:k\in S_i} p_i(x)$$

Throughput of link k

Probability of schedule i

Example: Assume $\alpha_{k,j} = \alpha$ for $k = 1, 2, 3$ and $j = 1, 2$.



$$\phi_2(x) \propto 2\alpha^3 x_1 x_2 x_3 + 2\alpha^2 x_2(x_1 + x_3) + 2\alpha x_2$$

Optimality of flow-aware CSMA

Theorem

The flow-aware CSMA algorithm is optimal: for any number of channels and any sets of conflicts graphs, it stabilizes the network for all traffic intensities inside the capacity region.

Sketch of proof:

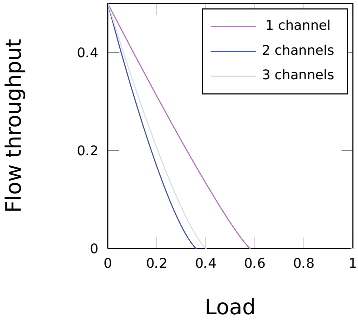
- When the number of flows is high, the algorithm behaves as max-weight: the algorithm selects schedules of high weight with high probability.
- Proof based on Foster's criterion.

Throughput performance

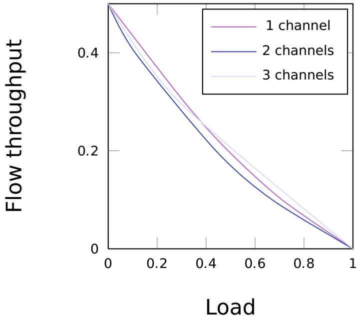
Homogeneous load

$$\alpha = 1$$

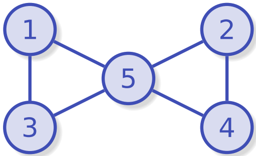
Standard CSMA



Flow-aware CSMA

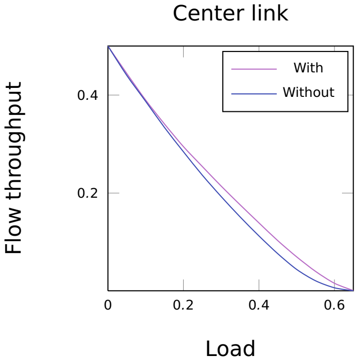


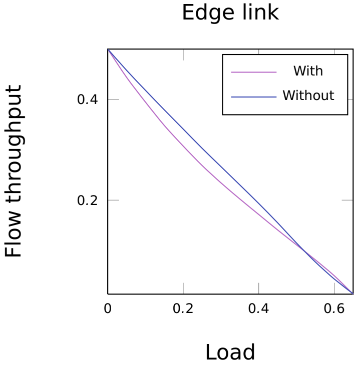
Time-scale separation



$\alpha = 1$

1 packet per flow





Outline

Model

Background

Standard CSMA

Flow-aware CSMA

Conclusion

Conclusion

A simple, distributed, asynchronous access scheme that is provably optimal.

A theoretical issue:

- Time-scale separation

Extensions of the model:

- Collisions
- Throughput performance